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TRANSVERSE VIBRATION OF A DAMAGED CIRCULAR ANNULAR PLATE WITH A FREE EDGE

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1. INTRODUCTION

The present study deals with the determination of the fundamental frequency of transverse vibration of a circular, annular plate simply supported or clamped at the outer edge and free at the inner edge (see Figure 1). It is assumed that the structural element, originally isotropic, experiences a damaging process in such a manner that the subdomain defined by $b \leqslant r \leqslant c$ acquires polarly anisotropic characteristics while the remaining portion conserves its original, virgin constitutive properties.

The fundamental eigenvalue is determined using the strain energy damage detection method as developed by Cornwell *et al.* [1] and following the modification proposed in reference [2]. It is important to point out that the fundamentals of the methodology were originally proposed by Stubbs *et al.* [3].

2. ANALYTICAL TREATMENT OF THE PROBLEM

Referring to the system shown in Figure 1 its transverse normal modes are governed by the well-known functional

$$J(W) = D\int_b^c \left[\left(W'' + \frac{W'}{\bar{r}}\right)^2 - 2(1-\upsilon)\frac{W'W''}{\bar{r}}\right]\bar{r}\,\mathrm{d}\bar{r}$$

$$+\int_c^a \left[D_r W''^2 + D_\theta\left(\frac{W'}{\bar{r}}\right) + 2D_r\upsilon_\theta \frac{W'W''}{\bar{r}}\right]\bar{r}\,\mathrm{d}\bar{r} - \rho h\omega^2\int_b^a W^2\,\bar{r}\,\mathrm{d}\bar{r} \tag{1}$$

subject to the boundary conditions

$$W''(b) + \frac{\upsilon}{b}W'(b) = 0, \quad W'''(b) + \frac{W''(b)}{b} - \frac{W'(b)}{b^2} = 0, \tag{2a, b}$$

$$W(a) = 0, \quad W''(a) + \frac{\upsilon_\theta}{a}W'(a) = 0 \quad \text{or} \quad W'(a) = 0. \tag{3a–c}$$

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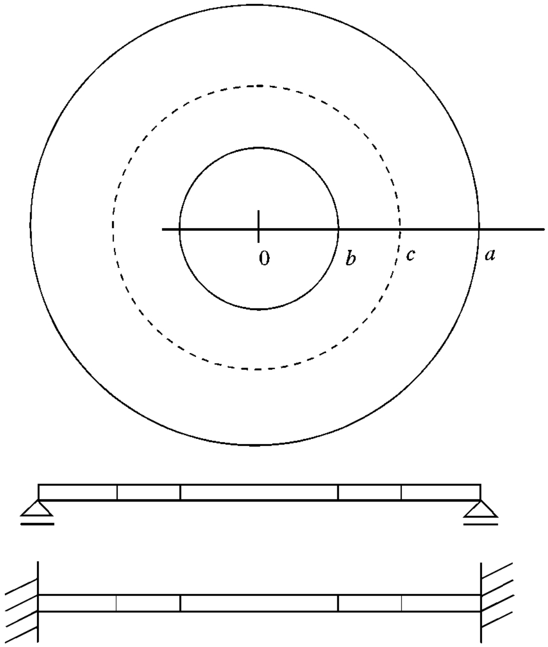

Figure 1. Vibrating structural system under study.

Introducing the dimensionless variable $r = \bar{r}/a$ and defining

$$r_b = b/a, \quad r_c = c/a \tag{4a, b}$$

one obtains, substituting in equations (1)–(3),

$$\frac{a^2}{D} J(W) = \int_{r_b}^{r_c} \left[\left(W'' + \frac{W'}{r} \right)^2 + 2(1-\upsilon) \frac{W'W''}{r} \right] r\,\mathrm{d}r + \int_{r_c}^{1} \left[\frac{D_r}{D} W''^2 + \frac{D_\theta}{D} \frac{W'^2}{r^2} + 2 \frac{D_r}{D} \upsilon_\theta \frac{W'W''}{r} \right] r\,\mathrm{d}r - \Omega^2 \int_{r_b}^{1} W^2 r\,\mathrm{d}r, \tag{5}$$

$$W''(r_b) + \frac{\upsilon}{r_b} W'(r_b) = 0, \quad W'''(r_b) + \frac{W''(r_b)}{r_b} - \frac{W'(r_b)}{r_b^2} = 0, \tag{6a, b}$$

$$W(1) = 0, \quad W''(1) + \upsilon_\theta W'(1) = 0 \quad \text{or} \quad W'(1) = 0 \tag{7a, b, c}$$

where

$$\Omega^2 = \frac{\rho h a^4}{D} \omega^2.$$

Substituting the approximating function

$$W \cong W_a = \sum_{j=1}^{N} C_j \varphi_j(r)$$

in the dimensionless functional (5) and applying Ritz' minimization conditions one obtains the following homogeneous linear system of equations:

$$\frac{a^2}{2D}\frac{\partial J}{\partial C_i} = \sum_{j=1}^{N}\left\{\int_{r_b}^{r_c}\left[\left(\varphi_j'' + \frac{\varphi_j'}{r}\right)\left(\varphi_i'' + \frac{\varphi_i'}{r}\right) - \frac{1-\upsilon}{r}(\varphi_j''\varphi_i' + \varphi_j'\varphi_i'')\right] r\,\mathrm{d}r\right.$$

$$+ \int_{r_c}^{1}\left[\frac{D_r}{D}\varphi_j''\varphi_i'' + \frac{D_\theta}{D}\frac{\varphi_j'\varphi_i'}{r^2} + \frac{D_r}{D}\upsilon_\theta\frac{\varphi_j''\varphi_i' + \varphi_j'\varphi_i''}{r}\right] r\,\mathrm{d}r$$

$$\left. - \Omega^2\int_{r_b}^{1}\varphi_j\varphi_i r\,\mathrm{d}r\right\} C_j = 0, \quad i = 1, \ldots, N. \tag{9}$$

The following co-ordinate functions are employed:

$$\varphi_j(r) = a_j r^{p+j-1} + b_j r^{j+2} + c_j r^{j+1} + 1, \tag{10}$$

where the coefficients are evaluated by substituting each polynomial in the governing boundary conditions with the exception of equation (2b) which was conveniently disregarded since the Rayleigh–Ritz method does not require satisfaction of the natural boundary conditions. The exponential parameter "p" constitutes Rayleigh's optimization variable.

3. NUMERICAL RESULTS

The numerical determinations were performed making $v = v_\theta = 0{\cdot}3$, $D_r/D = 1$ and $D_\theta/D = 0{\cdot}50$, 0·75, 1, 1·25 and 1·50 and taking $N = 7$.

In order to verify the accuracy of the approach it was first applied to the fully isotropic plate where very accurate results have recently been obtained following the exact method [4]. The agreement is remarkably good (see Table 1).

Table 2 depicts values of Ω_1 in the case of a damaged annular plate when the outer boundary is simply supported. The fundamental eigenvalues are obtained for $r_b = 0{\cdot}2$, 0·4 and 0·6 and $r_c = 0{\cdot}2, 0{\cdot}4, \ldots, 1$.

Table 3 deals with situation where the outer boundary is clamped.

The algorithmic procedure did yield an excellent rate of convergence in all the cases considered.

TABLE 1

Fundamental frequency coefficients of isotropic, annular plates

Outer BC		r_b 0·2	0·4	0·6
Simply supported	(1)	4·718	4·764	5·711
	(2)	4·7177	4·7640	5·7107
Clamped	(1)	10·409	13·603	25·674
	(2)	10·4080	13·6027	25·6742

Note: (1) Present approximate approach; (2) Exact solution [4].

TABLE 2

Fundamental frequency coefficients of vibrating isotropic–orthotropic plates (outer simply supported boundary)

D_θ/D	r_b	r_c 0·2	0.4	0·6	0·8	1
0·50	0·2	3·370	3·974	4·278	4·525	4·718
	0·4	—	3·244	4·029	4·462	4·764
	0·6	—	—	3·844	5·028	5·711
0·75	0·2	4·135	4·363	4·504	4·623	4·718
	0·4	—	4·085	4·412	4·616	4·764
	0·6	—	—	4·870	5·380	5·711
1	0·2	4·718	4·718	4·718	4·718	4·718
	0·4	—	4·764	4·764	4·764	4·764
	0·6	—	—	5·711	5·711	5·711
1·25	0·2	5·197	5·047	4·923	4·811	4·718
	0·4	—	5·343	5·092	4·907	4·764
	0·6	—	—	6·439	6·023	5·711
1·50	0·2	5·608	5·353	5·119	4·902	4·718
	0·4	—	5·854	5·399	5·046	4·764
	0·6	—	—	7·089	6·319	5·711

TABLE 3

Fundamental frequency coefficients of vibrating isotropic–orthotropic plates (outer clamped boundary)

D_θ/D	r_b	$r_c = 0·2$	0.4	0·6	0·8	1
0·50	0·2	9·132	9·188	10·224	10·383	10·409
	0·4	—	12·696	13·295	13·557	13·603
	0·6	—	—	25·052	25·558	25·674
0·75	0·2	9·827	10·150	10·317	10·396	10·409
	0·4	—	13·163	13·450	13·580	13·603
	0·6	—	—	25·366	25·616	25·674
1	0·2	10·409	10·409	10·409	10·409	10·409
	0·4	—	13·603	13·603	13·603	13·603
	0·6	—	—	25.674	25.674	25.674
1·25	0·2	10·911	10·660	10·500	10·422	10·409
	0·4	—	14·018	13·754	13·626	13·603
	0·6	—	—	25·977	25·732	25·674
1·50	0·2	11·352	10·904	10·590	10·435	10·409
	0·4	—	14·411	13·903	13·648	13·603
	0·6	—	—	26·275	25·790	25·674

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REFERENCES

1. P. Cornwell, S. W. Doebling and C. R. Farrar 1999 *Journal of Sound and Vibration* **224**, 359–374. Application of the strain energy damage detection method to plate-like structures.
2. P. A. A. Laura, C. A. Rossit, D. R. Avalos and H. A. Larrondo 1999 *Institute of Applied Mechanics*, *Bahía Blanca*, *Argentina*. Publication No. 99–33. Comments on "Application of the strain energy damage detection method to plate-like structures".
3. M. Stubbs, J. T. Kim and K. Topole 1992 *Proceedings of the ASCE 10th Structures Congress*, 543–546. An efficient and robust algorithm for damage localization in offshore platforms.
4. D. Vega, S. A. Vera, M. D. Sánchez and P. A. A. Laura 1998 *Journal of the Acoustical Society of America* **103**, 1225–1226. Transverse vibrations of circular annular plates with a free inner boundary.